Unit 3 - Test Review

Name Answer Key

Directions: Use the Remainder Theorem & Factor Theorem to answer the following

① Determine if $(x+9)$ is a factor of the function $f(x) = x^3 + 2x^2 - 51x + 108$.

a) Use synthetic Division

-9	1	2	-51	108
	↓	-9	63	-108
	1	-7	12	0

↑

R = 0, therefore $(x+9)$ is a factor

b) Use factor Theorem

$f(-9) = (-9)^3 + 2(-9)^2 - 51(-9) + 108$

$f(-9) = 0$, therefore is a factor.

② Determine if $(x+2)$ is a factor of $f(x) = x^3 + 2x^2 - 9x - 18$. If so, Find the other factors, then find the zeros.

-2	1	2	-9	-18
	↓	-2	0	18
	1	0	-9	0

↑ yes! Factor

$(x^2 - 9)(x+2) = 0$

$(x+3)(x-3)(x+2) = 0$

↑ Factored Form

therefore my zeros/Root/solutions $= -3, 3, -2 = x$

③ $f(x) = 4x^3 + 15x^2 - 63x - 54$ is $(x+6)$ a factor? Find other zeros!

-6	4	15	-63	-54
	↓	-24	54	54
	4	-9	-9	0

↑ therefore Factor

$(4x^2 - 9x - 9)(x+6) = 0$ → factored form

Bottom up method

$(4x^2 - 9x - 9)(x+6) = 0$

$x^2 - 9x - 36$

$(x - \frac{12}{4})(x + \frac{3}{4})$

$(x-3)(4x+3)(x+6) = 0$

therefore $\boxed{\text{Zeros} = 3, -6, -\frac{3}{4}}$

Use the fundamental theorem of algebra to determine the total number of roots.

a) $f(x) = x^4 - x^2 - 6$

4 total roots

b) $f(x) = -x^3 + 5x^2 + 12$

3 total zeros

c) $f(x) = x^5 + 7x^4 - 4x^3 - 3x^2 + 9x - 15$

5 total solutions

d) $f(x) = x^6 + x^5 - 3x^4 + x^3 + 5x^2 + 9x - 18$

6 total roots

Find all real zeros of the function.

a) $f(x) = 2x^3 - 3x^2 - 14x + 15$

$\uparrow q$ $\quad\quad \uparrow p$

$\dfrac{\text{Factors of } p}{\text{Factors of } q} : \dfrac{\pm 1, \pm 3, \pm 5, \pm 15}{\pm 1, \pm 2}$

so possible roots: $\pm 1 \pm 3 \pm 5 \pm 15$
$\pm 1/2 \pm 3/2 \pm 5/2 \pm \frac{15}{2}$

guess from list & check

$$\begin{array}{r|rrrr} 1 & 2 & -3 & -14 & 15 \\ & \downarrow & 2 & -1 & -15 \\ \hline & 2 & -1 & -15 & 0 \end{array}$$

so yes! Zero

$(2x^2 - x - 15)(x-1) = 0$

Factor by bottoms up

$(x^2 - x - 30)\ (x-1) = 0$

$(x - \frac{6}{2})(x + \frac{5}{2})\ (x-1) = 0$

Factored Form $\rightarrow (x-3)(2x+5)(x-1) = 0$

$X = 3, 1, \frac{-5}{2}$

b) $f(x) = 2x^4 - x^3 - 7x^2 + 4x - 4$

$\dfrac{\pm 1 \pm 2 \pm 4}{\pm 1 \pm 2}$ list $= \pm 1 \pm 2 \pm 4 \pm \frac{1}{2}$

guess & check

$$\begin{array}{r|rrrrr} 2 & 2 & -1 & -7 & 4 & -4 \\ & \downarrow & 4 & 6 & -2 & 4 \\ \hline & 2 & 3 & -1 & 2 & 0 \end{array}$$

yes it is a zero

$(2x^3 + 3x^2 - x + 2)(x-2) = 0$

Guess & check from list again

I'm guessing -2

$$\begin{array}{r|rrrr} -2 & 2 & 3 & -1 & 2 \\ & \downarrow & -4 & 2 & -2 \\ \hline & 2 & -1 & 1 & 0 \end{array}$$

$(2x^2 - x + 1)(x+2)(x-2) = 0$

Factor $\rightarrow (x^2 - x + 2)$

$(x - \frac{2}{2})(x + \frac{1}{2})$

$(x-1)(2x+1)(x+2)(x-2) = 0$

$X = 1, -2, 2, -\frac{1}{2}$ $\Rightarrow$ Zeros

Write a polynomial equation of least degree with the given zeros.

a) $-1, 2, 4$

$(x+1)(x-2)(x-4)$

$(x+1)(x^2 - 6x - 8)$

$x^3 - 6x^2 - 8x + x^2 - 6x - 8$

$x^3 - 5x^2 - 14x - 8 = f(x)$

b) $4, -\sqrt{5}, \sqrt{5}$

$f(x) = (x-4)(x+\sqrt{5})(x-\sqrt{5})$

$x^2 \cancel{-x\sqrt{5} + x\sqrt{5}} - 5$

$(x-4)(x^2-5)$

$x^3 - 5x - 4x^2 + 20$

$f(x) = x^3 - 4x^2 - 5x + 20$

c) $-1, 2, -3i$

Complex Conjugate Root Theorem says if $-3i$ is a root $+3i$ must also be a root

$(x+3i)(x-3i)(x+1)(x-2)$

$(x^2+9)(x+1)(x-2)$

$(x^2+9)(x^2-x-2)$

$x^4 - x^3 - 2x^2 + 9x^2 - 9x - 18$

$f(x) = x^4 - x^3 + 7x^2 - 9x - 18$