Answer Key

Quiz Tomorrow 12/7

Quiz Review

1. State in your own words "The Fundamental Theorem of Algebra"

The degree of the polynomial tell us how many solutions (roots) (zeros) there will be for that polynomial within the complex # system.

2. State in your own words "Complex Conjugate Root Theorem"

If a complex number is a root then the conjugate is also a root

Ex: If $3+2i$ is a root $3-2i$ is also a root

3. Why do we need the "rational root theorem"

allows us to generate a list of possible rational zeros to begin the guess and check process.

4. Find The Roots / Zeros

$\boxed{\text{* Set } f(x)=0 \text{ * the solve each Factor *}}$

a) $f(x)=(x-4)(x+2)(x-7)$

$x-4=0$ | $x+2=0$ | $x-7=0$

$x=4$ | $x=-2$ | $x=7$

b) $f(x)=(x^2-4)(x^2+4)$

$x^2-4=0$

$\sqrt{x^2}=\sqrt{4}$

$x=\pm 2$

$x^2+4=0$

$\sqrt{x^2}=\sqrt{-4}$

$x=\pm 2i$

$\boxed{\text{Remember } \sqrt{-1}=i}$

c) $f(x)=(x^2+8)(x-4)(x+7)$

$x^2+8=0$

$\sqrt{x^2}=\sqrt{-8}$

$x=\sqrt{8}\cdot\sqrt{-1}$

$=\pm 2i\sqrt{2}$

$x-4=0$

$x=4$

$x+7=0$

$x=-7$

d) $f(x)=(7x-4)(8x+2)$

$7x-4=0$

$7x=4$

$x=\frac{4}{7}$

$8x+2=0$

$8x=-2$

$x=-\frac{1}{4}$

⑤ State if the following given binomial is a factor.

a) $f(x) = 2x^3 + 9x^2 + 19x + 15$; $(x + \frac{3}{2})$

$-\frac{3}{2}$	2	9	19	15
	↓	-3	-9	-15
	2	6	10	0

yes!

Since the remainder = 0
$(x + \frac{3}{2})$ in a factor.
the zero of the polynomial is $x = -\frac{3}{2}$

b) $f(x) = 3x^3 + 9x^2 + 4x + 14$; $(x - 3)$

3	3	9	4	14
	↓	9	54	174
	3	18	58	188

No!

Since the remainder is $\neq 0$ (NOT)
$(x - 3)$ is NOT a factor

c) $f(x) = 3x^4 - 10x^3 - 24x^2 - 6x + 5$; $(x - 5)$

5	3	-10	-24	-6	5
	↓	15	25	5	-5
	3	5	1	-1	0

yes!

since the remainder is 0
$(x - 5)$ is a factor.

⑥ Find the zeros given a factor or zero.

a) $f(x) = x^3 - 2x^2 + x$; $(x - 1)$

1	1	-2	1
	↓	1	-1
	1	-1	0

$(x^2 - x)(x - 1) = 0$

GCF

$x(x - 1)(x - 1) = 0$

$x = 0$ $\quad x - 1 = 0$ $\quad x - 1 = 0$

$x = 0$ $\quad x = 1$ $\quad x = 1$

b) $3x^3 - 5x^2 - 47x - 15 = f(x)$; $(x + 3)$

-3	3	-5	-47	-15
	↓	-9	42	15
	3	-14	-5	0

$(3x^2 - 14x - 5)(x + 3) = 0$

$(x^2 - 14x - 15)(x + 3) = 0$

solve by Factoring "Bottoms" up

$(x^2 - 14x - 15)(x + 3) = 0$

$(x - \frac{15}{3})(x + \frac{1}{3})(x + 3) = 0$

$(x - 5)(3x + 1)(x + 3) = 0$

set = 0 and solve

$x = 5$ $\quad x = -\frac{1}{3}$ $\quad x = -3$

c) $f(x) = 3x^3 - 5x^2 - 3x + 5$; $x = 1$

$$\begin{array}{r|rrrr} 1 & 3 & -5 & -3 & 5 \\ & \downarrow & 3 & -2 & -5 \\ \hline & 3 & -2 & -5 & 0 \end{array}$$

$(3x^2 - 2x - 5)(x - 1) = 0$

$(x^2 - 2x - 15)(x - 1) = 0$

$(x - \frac{5}{3})(x + \frac{3}{3})(x - 1) = 0$

$(3x - 5)(x + 1)(x - 1) = 0$

$x = \frac{5}{3}$ $x = -1$ $x = 1$

List the Possible Rational Zeros of the polynomial

a) $f(x) = 3x^3 + 27$

$$\frac{p}{q} = \frac{\pm 1, \pm 3, \pm 9, \pm 27}{\pm 1, \pm 3} = \boxed{\pm 1, \pm 3, \pm 9, \pm 27, \pm \frac{1}{3}}$$

b) $f(x) = 7x^2 + 8x + 56$

$$\frac{p}{q} = \frac{\pm 1, \pm 2, \pm 4, \pm 7, \pm 8, \pm 14, \pm 28, \pm 56}{\pm 1, \pm 7}$$

list $\Rightarrow$ $\pm 1, \pm \frac{1}{7}, \pm 2, \pm \frac{2}{7}, \pm 4, \pm \frac{4}{7}, \pm 7, \pm 8, \pm \frac{8}{7}, \pm 14, \pm 28, \pm 56$

c) $f(x) = 2x^2 - 16$

factors of Constant / factors of L.C.

$$\frac{p}{q} = \frac{\pm 1, \pm 2, \pm 4, \pm 8, \pm 16}{\pm 1, \pm 2} \Rightarrow \pm 1, \pm \frac{1}{2}, \pm 2, \pm 4, \pm 8, \pm 16$$

Find the zeros of the polynomial given nothing

a) $f(x) = x^3 - 3x^2 - 4x + 12$

$\boxed{\text{list of possible zeros: } \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12}$

$$\begin{array}{r|rrrr} 2 & 1 & -3 & -4 & 12 \\ & \downarrow & 2 & -2 & -12 \\ \hline & 1 & -1 & -6 & 0 \end{array}$$

← Found a zero

$(x^2 - x - 6)(x - 2) = 0$

$(x - 3)(x + 2)(x - 2) = 0$

$x = 3$ $x = -2$ $x = 2$

b) $f(x) = x^3 + 3x^2 - 2x - 6$

$\boxed{\text{list of possible zeros: } \pm 1, \pm 2, \pm 3, \pm 6}$

Guess & Check from list

$$\begin{array}{r|rrrr} -3 & 1 & 3 & -2 & -6 \\ & \downarrow & -3 & 0 & 6 \\ \hline & 1 & 0 & -2 & 0 \end{array}$$

yes a zero!

$(x^2 - 2)(x + 3) = 0$

set each factor = 0 →

$x^2 - 2 = 0 \quad | \quad x + 3 = 0$

$x^2 = 2 \qquad x = -3$

$x = \pm\sqrt{2}$

c) $f(x) = x^3 - 3x^2 + x + 5$

$\boxed{\text{list: } \frac{p}{q} : \pm 1, \pm 5}$

$$\begin{array}{r|rrrr} -1 & 1 & -3 & 1 & 5 \\ & \downarrow & -1 & 4 & -5 \\ \hline & 1 & -4 & 5 & 0 \end{array}$$

Quadratic Formula → $(x^2 - 4x + 5)(x + 1) = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$

$= \frac{4 \pm \sqrt{-4}}{2}$

$= \frac{4 \pm 2i}{2} = \boxed{2 \pm i}$

$x + 1 = 0$

$x = -1$

$\boxed{\text{Zeros: } x = -1,\ = 2 + i,\ = 2 - i}$

3 solutions

d) $g(x) = 1x^4 + 4x^3 - x^2 + 16x - 20$

p ↓ q ↓

list of possible zeros: $\frac{p}{q} = \pm 1, \pm 2 \pm 4 \pm 5 \pm 10 \pm 20$

guess & check →

$$\begin{array}{r|rrrrr} 1 & 1 & 4 & -1 & 16 & -20 \\ & \downarrow & 1 & 5 & 4 & 20 \\ \hline & 1 & 5 & 4 & 20 & 0 \end{array}$$

$0 = (x^3 + 5x^2 + 4x + 20)(x - 1)$

Factor by grouping (4 terms)

$x^2(x + 5) \ 4(x + 5) \quad (x - 1) = 0$

$(x^2 + 4)(x + 5)(x - 1) = 0$

$x^2 + 4 = 0$

$x^2 = -4$

$x = \pm 2i \qquad x = -5 \qquad x = 1$

$\boxed{\text{Zeros } x = 1, -5, \pm 2i}$

e) $f(x) = 2x^3 - x^2 - 22x + 21$

list: $\pm \frac{1, 3, 7, 21}{1, 2}$

$$\begin{array}{r|rrrr} 1 & 2 & -1 & -22 & 21 \\ & \downarrow & 2 & 1 & -21 \\ \hline & 2 & 1 & -21 & 0 \end{array}$$

$(2x^2 + x - 21)(x - 1) = 0$

$(x^2 + x - 42)(x - 1) = 0$

$(x + \frac{7}{2})(x + \frac{6}{2})(x - 1) = 0$

$(2x + 7)(x + 3)(x - 1) = 0$

$x = -\frac{7}{2} \qquad x = -3 \qquad x = 1$

f) $f(x) = x^4 - 4x^3 - 7x^2 + 34x - 24$

list $\frac{p}{q} = \pm(1, 2, 3, 4, 6, 8, 12, 24)$

$$\begin{array}{r|rrrrr} 1 & 1 & -4 & -7 & 34 & -24 \\ & \downarrow & 1 & -3 & -10 & 24 \\ \hline & 1 & -3 & -10 & 24 & 0 \end{array}$$

$0 = (x^3 - 3x^2 - 10x + 24)(x - 1)$

* Use Rational Root Theorem again & synthetic Division again

$\frac{p}{q} = \pm 1, \pm 2 \pm 3 \pm 4 \pm 6 \pm 8 \pm 12 \pm 24$

$$\begin{array}{r|rrrr} 2 & 1 & -3 & -10 & 24 \\ & \downarrow & 2 & -2 & -24 \\ \hline & 1 & -1 & -12 & 0 \end{array}$$

Factor X → $(x^2 - x - 12)(x - 2)(x - 1) = 0$

$(x - 4)(x + 3)(x - 2)(x - 1) = 0$

$x = 4, \quad x = -3 \quad x = 2 \quad x = 1$

g) $3x^3 - 4x^2 - 2x + 3 = f(x)$

list: $\frac{p}{q} \quad \frac{\pm 1, \pm 3}{\pm 1, \pm 3} = \pm 1, \pm \frac{1}{3}, \pm 3$

$$\begin{array}{r|rrrr} 1 & 3 & -4 & -2 & 3 \\ & \downarrow & 3 & -1 & -3 \\ \hline & 3 & -1 & -3 & 0 \end{array}$$

Quadratic formula → $(3x^2 - x - 3)(x - 1) = 0$

$\frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(3)}}{2(3)}$

$x = \frac{1 \pm \sqrt{37}}{6}$

$x - 1 = 0$

$x = 1$

Zeros

$x = \frac{1 + \sqrt{37}}{6}, \quad x = \frac{1 - \sqrt{37}}{6} \quad x = 1$